$1/N_c$ Rotational Corrections to g_A in the NJL Model and Charge Conjugation

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We show that the $1/N_c$ rotational corrections to g_A, derived using the semiclassical quantization scheme in the NJL model, possess correct properties under charge conjugation.

Recently, the $1/N_c$ rotational corrections quantization scheme have been found [1,2] to provide a natural solution for the problem of strong underestimation of the axial-vector coupling constant g_A in the chiral quark soliton (Nambu–Jona-Lasinio) model in leading order. However, in a very recent paper Schechter and Weigel [3] state that the $1/N_c$ rotational corrections as they are proposed ref. [1] violate the G-parity reflection symmetry and accordingly they conclude that these corrections should exactly vanish. Actually they have raised an important question since the violation of G-parity reflection symmetry would indicate an inconsistency in the used scheme.

In this brief report we demonstrate that both the leading order term as well as the $1/N_c$ rotational correction as they are derived in ref. [2] possess correct symmetry properties under charge conjugation.

Similar to ref. [3] we choose to work with the G-parity transformation. Under this transformation the axial current $\bar{\psi}\gamma_\mu\gamma_5\tau^a\psi$ changes sign which leads to the well-known general relation

$$g_A[\bar{N}] = -g_A[N]\,, \tag{1}$$

between g_A for nucleon N and antinucleon $\bar{N}$.

In the NJL model the formula for g_A, derived [2] in the semiclassical quantization scheme, includes leading as well as next to leading order terms:

$$g_A = g_A^{(0)} + g_A^{(1)}\,. \tag{2}$$

The leading term is given by

$$g_A^{(0)} = -\frac{N_c}{9}\sum_{\text{occ. } m}\delta_{kb}\langle m|\gamma_0\gamma_5\gamma_k\tau_b\rangle m|\,, \tag{3}$$

whereas the next to leading contribution has more complicated structure[1]:

$$g_A^{(1)} = \frac{N_c}{9}\sum_{\substack{\text{occ. } m\\ \text{non-occ. } n}}\frac{i}{2\Theta}\frac{1}{\epsilon_n-\epsilon_m}\varepsilon^{akd}\langle m|\gamma_0\gamma_5\gamma_k\tau_d\rangle n|\langle n|\tau_a\rangle m|\,. \tag{4}$$

Here, Θ is the moment of inertia of the soliton. ϵ_m and $\rangle m|$ are the eigenvalues and eigenstates of the Dirac Hamiltonian $h(U)$:

$$h(U) = \gamma_0(-i\gamma^k\partial_k + MU^{\gamma_5} + m_0) \quad \text{and} \quad h(U) ranglem(U)| = E_m(U)\rangle m(U)| \tag{5}$$

In the case of nucleon (baryon number one solution) the occupied states inclide the Dirac sea and the valence level.

The meson field U has hedgehog symmetry

$$U(x) = \mathrm{e}^{iP(|\vec{x}|)(x^a\tau^a)/|\vec{x}|}\,. \tag{6}$$

which survives under G-parity transformation:

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[1]For simplicity, we use the non-regularized expressions. However, the regularization will not change our conclusions. More details about the derivation can be found in ref. [2].

$$GU(x)G^{-1} = U^\dagger(x). \tag{7}$$

This means that antinucleon is described by the hedgehog soliton with meson field $U^\dagger(x)$. Accordingly the relation 1 can be rewritten as

$$g_A[U^\dagger] = -g_A[U]\,. \tag{8}$$

It should be noted that both charge conjugation and G-parity transformation can be used to obtain antinucleon state. However, the G-parity transformation is more convenient since it explicitly preserves both the isospin and the hedgehog ansatz whereas in the case of the charge conjugation one should use additionally the invariance of the hedgehog solution 6 under SU(2)-isorotation. It should be also stressed that both transformations lead to one and the same result 8.

Using the identity

$$h(U^\dagger) = -(\gamma_0\gamma_5)h(U)(\gamma_0\gamma_5)^{-1}\,, \tag{9}$$

it easy to see that following relations are valid:

$$\rangle m(U^\dagger)| = \gamma_0\gamma_5\rangle m(U)| \qquad \text{and} \qquad \epsilon_m(U^\dagger) = -\epsilon_m(U)\,. \tag{10}$$

Let us start from the leading contribution $g_A^{(0)}$ 3. First, from 10 we have

$$m(U^\dagger)\gamma_0\gamma_5\gamma_k\tau_b\rangle m(U^\dagger)| = \langle m(U)|\gamma_0\gamma_5\gamma_k\tau_b\rangle m(U)|\,. \tag{11}$$

Second, using the identity

$$\sum_{\text{occ. } m(U)} \delta_{kb}\langle m(U)|\gamma_4\gamma_5\gamma_k\tau_b\rangle m(U)| = -\sum_{\text{non-occ.} m(U)} \delta_{kb}\langle m(U)|\gamma_4\gamma_5\gamma_k\tau_b\rangle m(U)|\,, \tag{12}$$

and also the fact that the occupied states of $h(U)$ correspond to non-occupied states of $h(U^\dagger)$ we find in agreement with 8:

$$g_A^{(0)}[U^\dagger] = -g_A^{(0)}[U]\,. \tag{13}$$

For the rotational correction $g_A^{(1)}$ in the case of antinucleon using again relations 10 we have

$$\begin{aligned}
&\sum_{\substack{\text{occ. } m(U^\dagger)\\ \text{non-occ. } n(U^\dagger)}} \frac{1}{\epsilon_n(U^\dagger)-\epsilon_m(U^\dagger)}\langle m(U^\dagger)|\gamma_0\gamma_5\gamma_k\tau_d\rangle n(U^\dagger)|\langle n(U^\dagger)|\tau_a\rangle m(U^\dagger)| \\
&= -\sum_{\substack{\text{non-occ. } n(U)\\ \text{occ. } m(U)}} \frac{1}{\epsilon_n(U)-\epsilon_m(U)}\langle m(U)|\gamma_0\gamma_5\gamma_k\tau_d\rangle n(U)|\langle n(U)|\tau_a\rangle m(U)|\,.
\end{aligned} \tag{14}$$

Further we make use of the properties of the matrix elements under the interchange $m \longleftrightarrow n$

$$\langle m|\gamma_0\gamma_5\gamma_k\tau_d\rangle n|\langle n|\tau_a\rangle m| = -\langle n|\gamma_0\gamma_5\gamma_k\tau_d\rangle m|\langle m|\tau_a\rangle n|\,, \tag{15}$$

to get the final result:

$$g_A^{(1)}[U^\dagger] = -g_A^{(1)}[U]\,. \tag{16}$$

One concludes that the next to leading order term shows the same G-parity properties as the leading one and both agree with 8.

Actually the expression 4, studied above, are derived in a formalism based on the path integral approach as it is presented in ref. [2]. To be more particular, in the derivation of the $1/N_c$ rotational corrections to g_A using semiclassical quantization not-commuting collective operators appear and the final result depends on the ordering of these operators. Our scheme involves time-ordering of collective operators which follows from the path integral without any ambiguity, applicable of course to both nucleon and antinucleon. In this respect it is not surprising that

G-parity properties are accurately described. In contrast to us, the scheme of Wakamatsu and Watabe [1] uses a different ordering whose theoretical origin is not justified. Hence their formulae are different from the ones of ref. [2] and in particular, as it has been shown in ref. [3] it violates the G-parity reflection symmetry.

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